Sensitivity analysis of the Hoek-Diederichs rock mass deformation modulus estimating formula

P. Ván
Department of Theoretical Physics, Research Institute for Particle and Nuclear Physics, HAS, Budapest, Hungary and
Bergen Computational Physics Laboratory, BCCS, Bergen, Norway

B. Vásárhelyi
Dept. Engng. Mat. & Engng. Geol., Budapest University of Technology and Economics, Budapest, Hungary

ABSTRACT: There are several empirical formulas in the literature to determine the rock mass modulus (E_{rm}) from rock mass classification (ie. from RMR, Q, GSI or RMi values). Recently, Hoek and Diederichs (2006) examined a large set of field measurement data and suggested new formulas to estimate the deformation modulus of the rock mass (E_{rm}), using the Geological Strength Index (*GSI*) and the disturbance factor (*D*). Their formula is based on the observation that a sigmoid function can be fitted well, both for the usual test data of empirical estimation formulas and for the larger data set of measurements. Determination both the *GSI* and the disturbance factor (*D*) are mostly subjective, thus it is important to know how sensitive E_{rm} measurements using the published Hoek-Diederichs equations are as a consequence of this subjectivity. The goal of this paper is to determine the sensitivity of these equations.

1 INTRODUCTION

All in situ deformation tests are expensive, often difficult to measure and time-consuming. Because of this, the modulus of deformation is often estimated indirectly from observations of relevant rock mass classification systems – such as Rock Mass Rate (RMR, introduced by Bieniawski, 1973), Tunneling Quality Index (Q, see Barton et al. 1974), Rock Mass index (RMi, developed by Palmström, 1995) and Geological Strength Index (GSI, published firstly Hoek et al. 1995). Palmström and Singh (2001) compared these empirical relationships, determining the validity of them.

Recently, Hoek and Diederichs (2006) examined a large set of field measurement data and suggested new formulas to estimate the deformation modulus of the rock mass (E_{rm}), using the Geological Strength Index (*GSI*) and the disturbance factor (*D*). Their formula is based on the observation that a sigmoid function can be fitted well, both for the usual test data of empirical estimation formulas (Serafim & Pereira, 1983; Bieniawski, 1978; and Stephens & Banks, 1989) and for the larger data set of Chinese and Taiwanese measurements (Hoek & Brown, 1997). Sigmoid functions are common in several areas of technology and physics (e.g. the Fermi-Dirac distributions in quantum ideal gases).

Determination both the Geological Strength Index (*GSI*) and the disturbance factor (*D*) are very subjective. Recently, Edelbro et al. (2007) published their results, determining the different rock mass values by 11 independent participants, getting high differences between the minimum and the maximum values. The disturbance factor can be more difficult to determine exactly – up to now it is not standardized. This is why so important to know how sensitive E_{rm} measurements using the published Hoek-Diederichs equations are as a consequence of this subjectivity (Hoek & Diederichs, 2006).

The published analysis method can be used to determine the sensitivity for the other applied rock mass deformation modulus empirical equations (summarized and analyzed them e.g.Palmström & Singh 2001), as well.

2 HOEK-DIEDERICHS FORMULAS

The Hoek-Diederichs formulas (Hoek & Diederichs, 2006) are based on the value of the Geological Strength Index (*GSI*) and the disturbance factor (*D*), which factor was firstly introduced by Hoek et al. (2002). Hoek et al (2002) and Hoek and Diederichs (2006) give several examples to estimate the disturbance in particular practical situations, however it is very subjective: it is assigned a value between 0 (undisturbed) and 1 (fully disturbed).

The introduced formula of Hoek and Diederichs (2006) calculates the deformation modulus from the *GSI* value and *D* factor as:

$$\mathrm{E_{rm}}(MPa) = 100{,}000 \frac{1 - D/2}{1 + e^{(75+25D-GSI)/11}} \tag{1}$$

or if the deformation modulus of the intact rock (E_i) is known, equation (1) can be modified to:

$$\mathrm{E_{rm}}(MPa) = E_i \left(0.02 + \frac{1 - D/2}{1 + e^{(60+15D-GSI)/11}} \right) \tag{2}$$

The simplified (Eq. 1) and the more comprehensive Hoek-Diederichs equation (2), are shown in Figure 1 and 2, respectively.

Using the two formulas the estimated deformation moduli are not the same, they depend on the deformation modulus of the intact rock – the ratio of the two results in case of low GSI values can be large. For different disturbance factors (D = 0, 0.5 and 1), these differences are plotted on Figures 3-5, respectively.

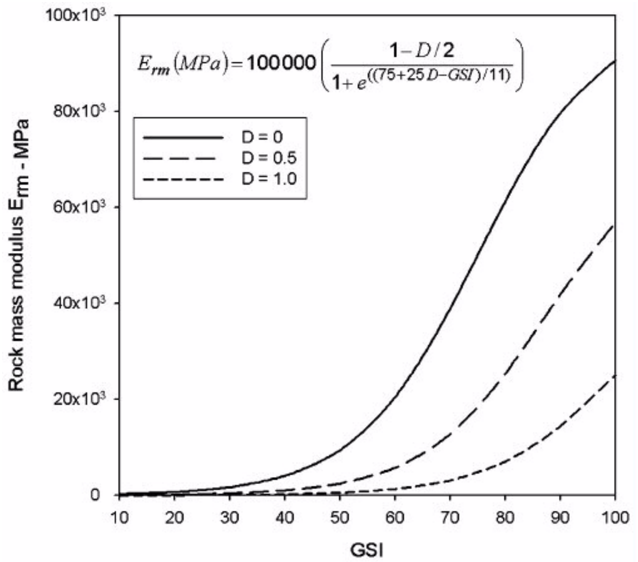

Figure 1. Simplified Hoek-Diederichs equation (1) for empirical estimates of rock mass deformation modulus based on *GSI* and *D* only.

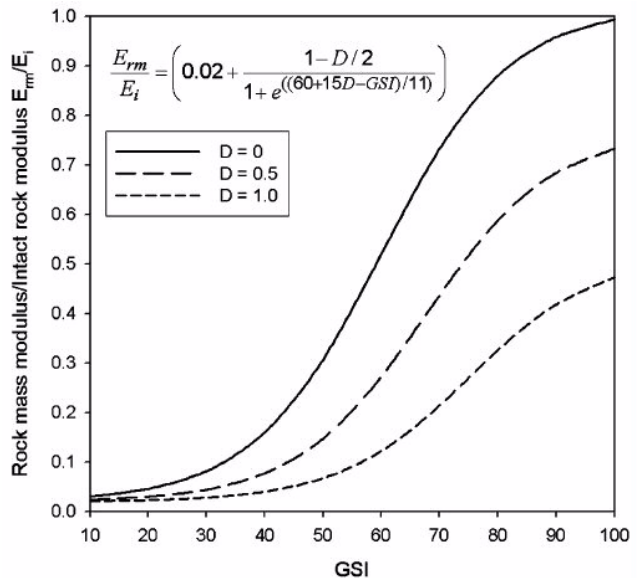

Figure 2. Hoek-Diederichs equation (2) for empirical estimates of rock mass deformation modulus based on *GSI*, *D* and intact rock modulus E_i.

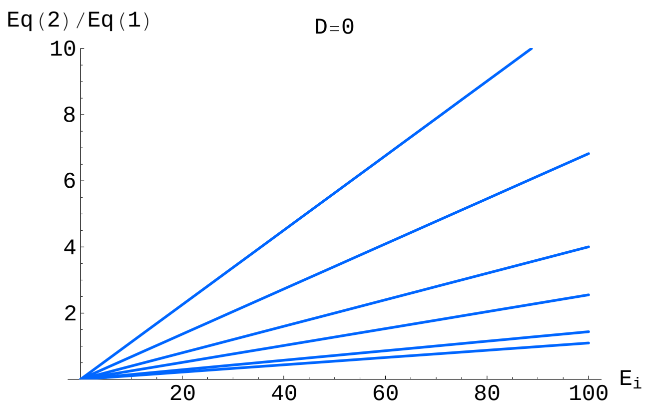

Figure 3. The ratio of Eq(2) and Eq(1) as a function of deformation modulus of intact rock in case of different GSI values (D = 0). GSI =(10, 20, 40, 60, 80, 100) from above, respectively.

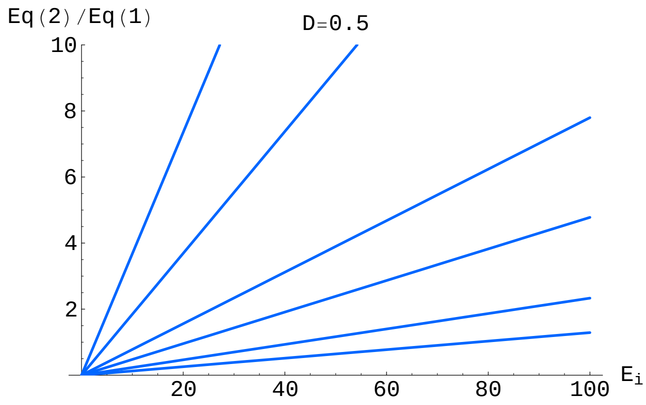

Figure 4. The ratio of Eq(2) and Eq(1) as a function of deformation modulus of intact rock in case of different GSI values (D = 0.5) GSI =(10, 20, 40, 60, 80, 100) from above, respectively.

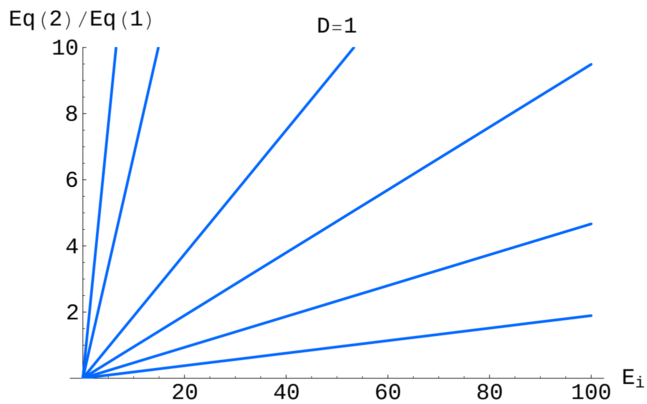

Figure 4. The ratio of Eq (2) and Eq(1) as a function of deformation modulus of intact rock in case of different GSI values (D = 1), GSI =(10, 20, 40, 60, 80, 100) from above, respectively.

3 APPLIED SENSITIVITY ANALYSIS

The sensitivity of a function *f* regarding the uncertainties of the variables can be characterized by the formula commonly known as propagation of error (Bronstein & Semendjajew, 2004).

Let us suppose that *f* is a real functionwhich depends on *n* variables $x_1, x_2, \ldots x_n$ and the uncertainty of each we can calculate the uncertainty Δf of f that results from the uncertainties of the variables:

$$\Delta \mathrm{f} = \left(\sum_{\mathrm{i}=1}^{\mathrm{n}} \left(\frac{\partial f}{\partial x_i} \Delta x_i \right)^2 \right)^{\frac{1}{2}} \tag{3}$$

4 SENSITIVITY OF THE H-D EQUATIONS

The sensitivity of different empirical formulas to parameter uncertainty is an important factor for a designer. To establish good empirical formulas one should have some sense on the effect of variations in the input parameters to judge the acceptability of the design. In this note we analyze the above formulas from this point of view, giving some practical tools to enable rapid sensitivity analyses.

In estimating the sensitivity, it was assumed that the variables are uncorrelated, therefore, one can apply equation (3) (Bronstein & Semendjajew, 2004). Assuming that the sensitivity in the disturbance factor D is ΔD and in the GSI it is ΔGSI, one can get:

$$\Delta E_{rm}(MPa) = \sqrt{\left(\frac{1}{11(1+A)}\Delta GSI\right)^2 + \left(\left(\frac{1}{D-2} - \frac{25}{11(1+A)}\right)\Delta D\right)^2} \quad (4)$$

where

$$A = e^{(GSI-75-25D)/11}$$

The relative sensitivity for the simple Hoek-Diederichs criteria of equation (1) is plotted in the case of $\Delta D = 0.05$ and $\Delta GSI/GSI = 0.05$ in Figure 6 for disturbance values of $D = 0$, 0.5 and 1. One can see that the sensitivity in the rock mass modulus is between 15-35% and strongly depends on the GSI value. There is a peak in the sensitivity between GSI values of 60 and 80. Figure 7 shows the corresponding absolute sensitivity according to equation (4). The law of Gauss applied to the modified Hoek-Diederichs criteria (Eq. 2) gives

$$\Delta E_{rm}(MPa) = \left(1 - \frac{0.02E_i}{E_{rm}}\right) \text{x} \quad (5)$$

$$\text{x}\sqrt{\left(\frac{1}{11(1+A)}\Delta GSI\right)^2 + \left(\left(\frac{1}{D-2} - \frac{15}{11(1+A)}\right)\Delta D\right)^2}$$

where

$$A = e^{(GSI-60-15D)/11}$$

The relative sensitivity estimated by equation (4) is plotted for $\Delta D = 0.05$ and $\Delta GSI/GSI = 0.05$ in Figure 8 for values of $D = 0.0$, 0.5 and 1.0. The sensitivity in the rock mass modulus is between 0.5-22 % and again, it strongly depends on the GSI value. The peaked property is even more apparent in this case, with the greatest sensitivity occurring for GSI values between 40 and 60. Figure 9 shows the corresponding absolute sensitivity according to equation (5).

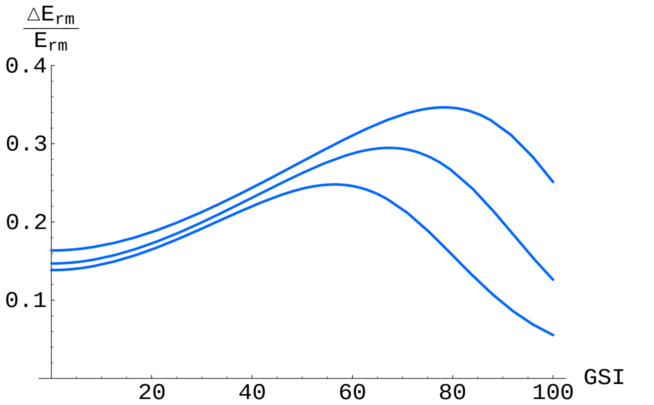

Figure 6 Relative sensitivity of the simple Hoek-Diederichs function (Eq. 1) as a function GSI, in case $\Delta D = 0.05$, $\Delta GSI/GSI = 0.05$ if $D = 0$, 0.5 and 1 (from below).

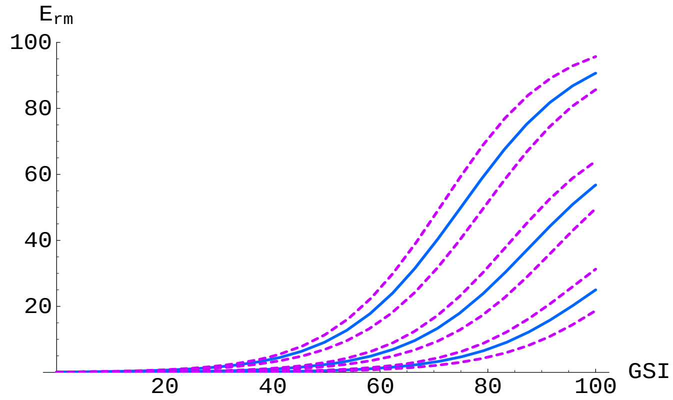

Figure 7 Absolute sensitivity of the simple Hoek-Diederichs function (Eq. 1) as a function GSI, in case $\Delta D = 0.05$, $\Delta GSI/GSI = 0.05$ if $D = 0$, 0.5 and 1 (from below). The dashed lines around the solid ones denote the sensitivity bar levels.

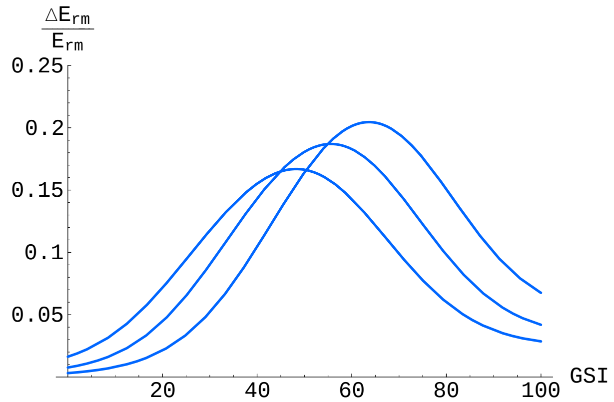

Figure 8 Relative sensitivity of the modified Hoek-Diederichs equation (Eq.2) as a function GSI, in case $\Delta D = 0.1$ and $\Delta GSI = 0$ if $D = 0$, 0.5 and 1 (from below at left).

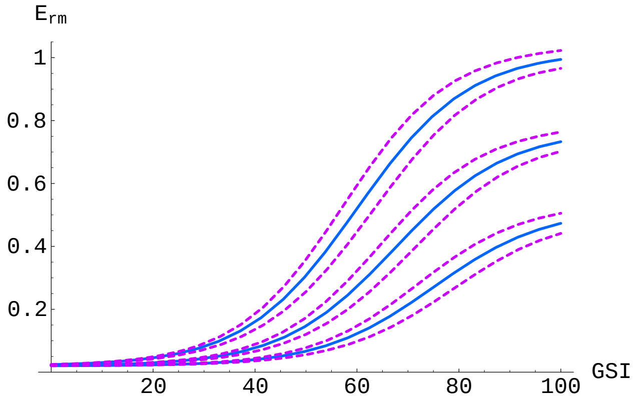

Figure 9 Absolute sensitivity of the modified Hoek-Diederichs equation (Eq. 2) as a function *GSI*, in case $\Delta D = 0.05$, $\Delta GSI/GSI = 0.05$ if $D = 0$, 0.5 and 1 (from below). The dashed lines around the solid ones denote the sensitivity bar levels.

5 CONCLUSIONS

Using the Hoek-Diederichs equations, the rock mass deformation modulus can be determined if the Geological Strength Index (GSI) and the disturbance factor (D) are known. The determination of each parameter is subjective, and thus, to know the sensitivity of these equations is very important. Using the formula of Gauss, the sensitivity of the equations was analyzed for $\Delta D = 0.05$ and $\Delta GSI/GSI = 0.05$ for $D = 0$, 0.5 and 1. It was shown, that in case of simple H-D equation the sensitivity in the rock mass modulus is between 15-35 % and for the modified H-D equation it is between 0.5-22 %. In both cases the sensitivity strongly depends on the GSI value.

ACKNOWLEDGMENT

The authors are acknowledge the financial support for the Bolyai Scolarship and the Hungarian National Research Foundation (OTKA T048489, K60768 and D48645) and by the EU-I3HP project.

REFERENCES

Barton, N., Lien, R. & Lunde, J. 1974. Engineering classification of rock masses for design of rock support. *Rock Mech*. **6**: 189-236.
Bieniawski, Z.T. 1973. Engineering classification of jointed rock masses. *Trans. S. African Inst. Civ. Engrs*., **15**335 -344.
Bieniawski, Z.T. 1978. Determining rock mass deformability – experience from case study. *Int. J. Rock Mech. Min. Sci. Geomech. Abst.,* **15**237 -247.
Bronstein, I.N. & Semendjajew, K.A. 2004. *Handbook of Mathematics*, Springer, Berlin (4th edition).
Edelbro, C., Sjöberg, J. & Nordlund, E. 2007. A quantitative comparison of strength criteria for hard rock masses. *Tunneling & Underground Space Techn.* **22**57 -68.
Hoek, E. & Brown, E.T. 1997. Practical estimates or rock mass strength. *Int. J. Rock Mech. Min. Sci. Geomech. Abst.,* **34**: 1165-1186.
Hoek, E., Carranza-Torres, C. & Corkum, B. 2002. Hoek-Brown failure criterion – 2002 Edition. *Proc. 5. North American Rock Mech. Conf.* **1**: 267-271.
Hoek, E. & Diederichs, M.S. 2006. Empirical estimation of rock mass modulus. *Int. J. Rock Mech. Min. Sci.* **43**203 -215.
Hoek, E., Kaiser, P.K. & Bawden, W.F. 1995. *Support of underground excavations in hard rock*. Balkema.
Palmstöm, A. 1995. RMi - a rock mass characterization system for rock engineering purposes. PhD These. (www.rockmass.net)
Palmström, A. & Singh, R. 2001. The deformation modulus of rock masses – comparisons between in situ tests and indirect estimates. *Tunneling & Underground Space Techn.* **16** 115 -131.
Serafim, J.L. & Pereira, J.P. 1983. Consideration of the geomechanical classification of Bieniawski. *Int. Symp. Engng Geol. & Underground Constructions.* **1/II**: 33-44
Sonmez, H. & Gokeceoglu, C. 2006. Discussion of the paper by E. Hoek and M.S. Diederichs "Empirical estimation of rock mass modulus". *Int. J. Rock Mech. Min. Sci*. **43** 671-676.
Stephens, R.E. & Banks, D.C. 1989. Moduli for deformation studies of the foundation and abutments of Portugues Dam – Puerto Rico. *Proc. 30th US Symp.* 31-38.